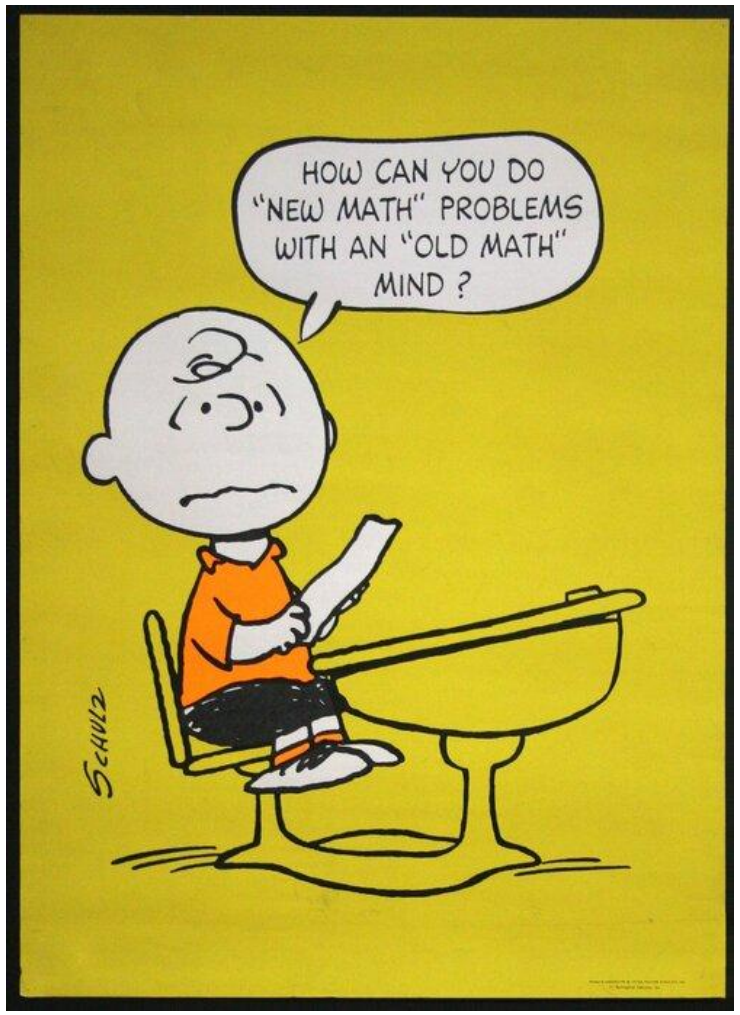


Category Theory – my struggle so far

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What is it, what is it for?

Well that was my first problem.

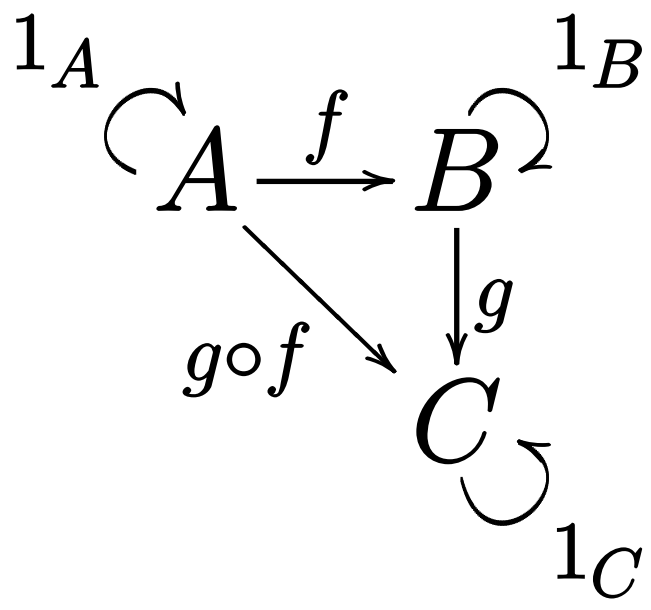
“It is difficult to preview the main theorems in category theory before developing fluency in the language needed to state them” – *Category Theory in Context by Emily Riehl.*

More hopefully in Wikipedia:

Category theory is a general theory of mathematical structures and their relations.

Arrow diagrams

This is what a category consists of! Normally don't show the identity or composed morphisms unless they are relevant



A, B, C, … are objects
f, g, h, … are morphisms
$g \circ f: A \to C$ is a composition
The identity morphisms 1_A, 1_B, 1_C, …
always exist $f \circ I_A = f = I_B \circ f$
Morphisms are associative
$h \circ (g \circ f) = (h \circ g) \circ f$

Some Simple Examples

Set: objects are sets which form the domains and codomains of functions which are the morphisms.

Poset: partially ordered set. The relation $\geq$ is the morphism and if $A \geq B$ and $B \geq A$ then $A = B$

Mat: the objects are the natural numbers giving a size, the morphisms are matrices with row size of the source and column size of the target. Identities are identity matrices.

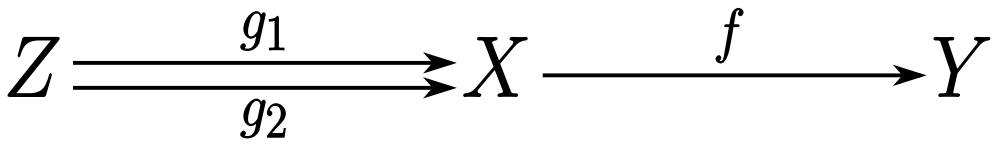
Pno: Peano numbers. Represented by a single object. The identity morphism represents 0 and a single ‘free’ morphism back to itself operates as the successor function succ.

Lots of new or reused words and phrases

Second big problem, for me at least. “This is quite an interesting situation so gets a name” – *The Joy of Abstraction by Eugenia Cheng*

morphism: auto-, bi-, endo-, epi- , homeo-, homo-, mono, iso-,
Duals: co -limit , -product, -cone, -unit, -complete, -sheave, pullback, pushout, covariant, contravariant, left/right adjoint, monic, epic, section, retraction , categorification, decategorification, initial, terminal, functor, arrow, comma, monad, monoid, groupoid, confluent, adjunct, universal cones, diagram chase, inverse, pointed, monotone, poset, preset, toset, natural transformation, action preserving, module, projection, equalizer, restriction, hom-set, forgetful, free, …

Third problem thinking categorically



f is a monomorphism (monic) if

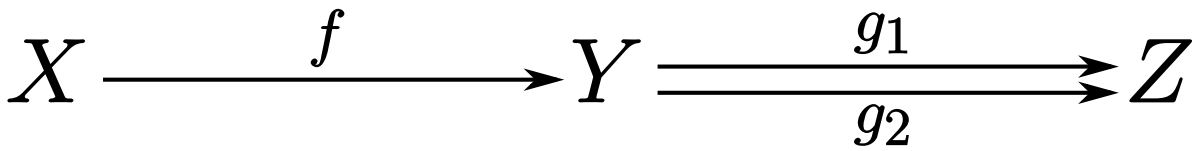
$f \circ g_1 = f \circ g_2$ implies $g_1 = g_2$

For a Set category with functions as morphisms it implies the function is injective

Morphisms are not always functions. However this definition of monic can be applied elsewhere as it only refers to concepts in category theory.

Duals

However category theory gives (should give?) a reward for the work. Duality is an interesting and important one. Epics are the dual of monics got by reversing the arrows.



f is a epimorphism (epic) if
$g_1 \circ f = g_2 \circ f$ implies $g_1 = g_2$

For sets it implies the function is surjective. Who imagined injective and surjective were such close duals?

Functors

Samuel Eilenberg and Saunders MacLane – who founded category theory in 1945:

“It should be observed first that the whole concept of a category is essentially an auxiliary one; our basic concepts are essentially those of a functor and of a natural transformation.”

The origins go back to the 1930’s in Poland and to Emmy Noether in Germany.

Problem n+1. We’re just about seeing the foothills!

Up the ladder

A selection or groups of objects in a category can often be treated as objects in another category. Morphisms can be treated as objects in another category. A category can therefore be enhanced with information from categories describing parts of it.

Categories can even be treated as objects. Functors are mappings between categories so the morphisms are respected and therefore can show relationships between quite different areas.

All this recursion counts as problem++ for me!

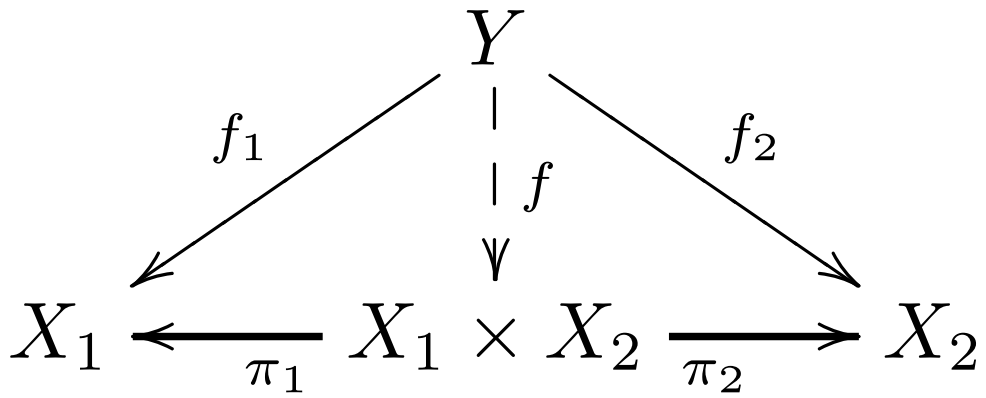
Conclusion

Category theory is being used more and more as the language and methodology for linking up between different areas of mathematics.

Unfortunately that means looking at things in a different way, which can be more difficult than just learning something new. It is becoming a new higher layer in the foundations above set theory - able to do most of the things set theory does, but closer to the algebraic and typed way other areas are developed.

As to the future, I see it as great for AI. I'm not sure exactly how I'll fare but it'll certainly keep me well occupied for a while! ☺

To end with – A definition of a Product



$X_1 \times X_2$ with morphisms π_1 and π_2 is a product if given any Y with morphisms f_1 and f_2 there is a unique morphism f as in that diagram so the triangles commute ($f_1 = \pi_1 \circ f,\ f_2 = \pi_2 \circ f$).

For a category of sets the count of elements in $X_1 \times X_2$ is the product of the count of elements in X_1 and X_2.

And the dual, the co-product for sets is … drumroll … the sum!